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Graph Analysis for Thermal Networks

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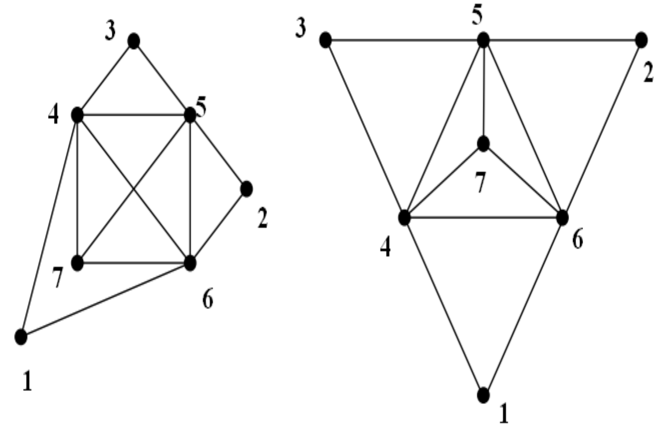
Graph Analysis for Thermal Networks

Algorithms of comparison of network graphs by coincidence of vertex sets with the same eccentricity or weight with respect to the centroid are considered. The code fragments of the proposed techniques are given. The comparison method can be used in simulators for remote training of operators and engineers of thermal networks, where the developed scheme is compared with the test one.



Planar isomorphic graphs
with matching numbering of all vertices

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THE METHOD OF ECCENTRICITIES COMPARISON

To identify the isomorphism of such graphs in this paper we propose to use the method of comparing the sets of labeled vertices in with the same eccentricity.The first area for comparison is *the center* of the graph. The eccentricity of these vertices is by definition equal to the radius ε= r.

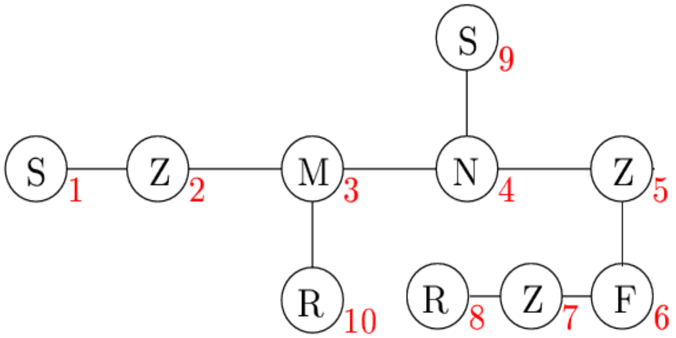
$M_r(G)$– set of vertex labels for the center of the graph G(zero-level area).

To select the vertexes of subsequent levels:

$$M_{r+k}(G), k = 0, \dots, d - r$$

The condition of isomorphism :

$$M_{r+k}(G') = M_{r+k}(G), k = 0, \dots, d - r$$



Example of a graph with weighted vertices

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Example

Here is an example of calculation using operators of the Maple computer mathematics system from the networks package.

Setting vertices of an undirected graph :

V:= {$1..n}:

Setting edges of an undirected graph :

E:={{1,2},{2,3},{3,4},{4,5},{5,6},{7,6},{7,8},{4,9},{3,10}}:

Creating a graph *G(V,E)* (Fig.1):

G:=graph(V, E):

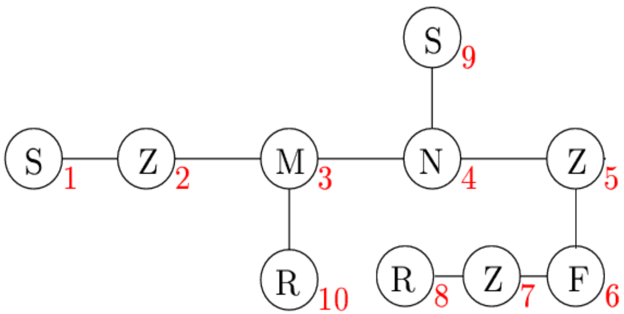

Fig.1 Undirected graph with weighted vertexes

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Getting the adjacency matrix (Fig.2) executed by the **adjacency** command

To determine the distance between vertexes :

s:=allpairs(G),**s[i,j] -** the distance between vertices i and j.

Receive eccentricities of the vertices :

for i to n do

  W[i]:=max(seq(s[i,u],u=1..n)):

od:

Finding the radius of a graph :

r:=min(seq(W[i],i=1..n));

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Fig.2 Adjacency matrix of a graph G

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The NM matrix contains designations for elements of the heat network :

S - input, Z - gate, M - acid gauge, N - pump, F — filter, R - output.

NM:=[S, Z , M, N,Z, F ,Z,R,S, R]:

Selecting vertices of the graph center(Fig.3):

```
for j to n do
  if W[j]=r then print(j,NM[j]) end:
end;
```

To get subsequent levels :

```
for j to n do if W[j]=r+N
  then print(j,NM[j]) end:
end:
```

N — level number

Fig.3 Level 0

Fig.4 Level 1

Fig.5 Level 2

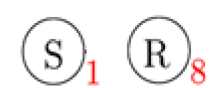

Fig.6 Level 3

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Visual splitting of vertexes into levels Fig.7

Note that the set II contains two vertices 2 and 7 with the same Z marks .

In general, this can cause some difficulties in identifying the graph. This is because the numbers j that the program produces in the print(j, NM[j]) statement are conditional. For compared graphs in general, they are different even if the graphs are isomorphic.

However, the probability of erroneous identification will increase with the increase in the "radial thickness" of the graph, equal to the difference in diameter and radius.

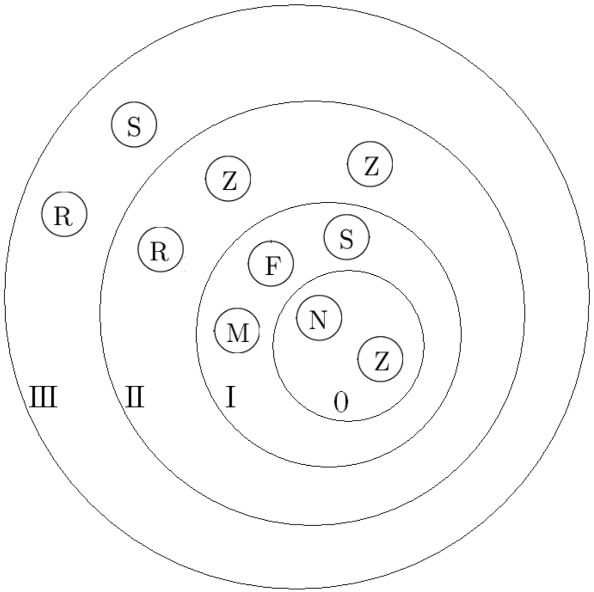

Fig.7 Splitting the set of vertices of a graph into regions of equal eccentricity

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If now this algorithm is applied to the graph in Fig.8, and it is obviously isomorphic to the graph in Fig.1), differing from it only in the numbering of vertices, then all four lists of areas 0,1,2,3 will be the same. This confirms that the graphs are isomorphic.

0	1	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0
0	0	0	1	0	1	0	0	0	1
0	0	0	0	1	0	1	0	1	0
0	0	0	0	0	1	0	1	0	0
0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0	0	0

Adjacency matrix of an isomorphic graph

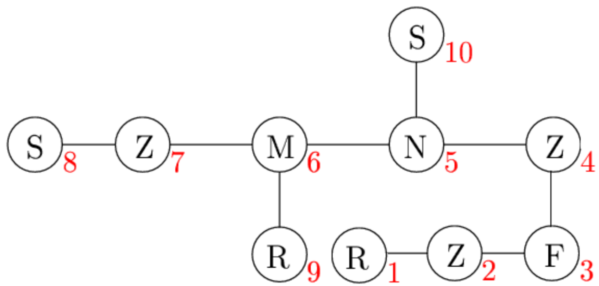

Fig.8 Graph 2 with marked vertexes

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In practice, there are graphs of more complex structure, for which the proposed algorithm is also applicable (Fig. 9). The alternative method with centroid extraction is applicable only to trees. In particular, this approach, which also distinguishes vertex groups by analogy with Fig. 7 regardless of their numbers, but only by the graph structure, can serve as a test of the proposed method.

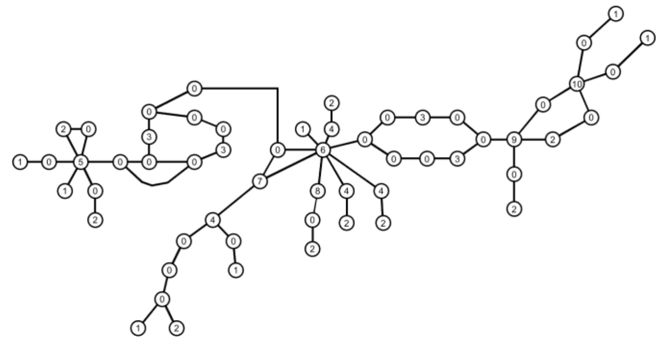

Fig. 9 Graph with cycles

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The calculation of the centroid

```
> DL:=maxdegree(G):
> V:=Vector(DL):
```

The cycle to calculate the weights of the vertices

```
> for i to n do
  H:=duplicate(G):
# Successive removal of vertices of copies of G
>   H:=delete(i,H):
>   Di:=vdegree(i,G);
# List of vertices of each branch:
>   K:=components(H):
>   for j to nops(K) do
# The weight of the branch
 V[j]:=nops(edges(induce(K[j],G)))+1; >   od:
# Top weight
> W[i]:=max(seq(V[k],k=1..Di));
> od:
# Array of weights
> W:=convert(W,array);
 #Centroid:
> minW:=min(seq(W[i],i=1..n));
# Lowest vertex weight
> C:={}:
> for i to n do
> if W[i]=minW then
> C:=C union {i};
> fi;
> od:C;
```

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Example of a field of use: simulators for training operators and engineers of heat networks (Fig.10)
Advantages of the algorithm:

- Easy to program
- Does not require a lot of resources
- Compact information about graph schemas

Fig.10 Stand of the simulator

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Thank you for attention!

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