Matlab Application to Calculate Natural Oscillations of Beam Systems

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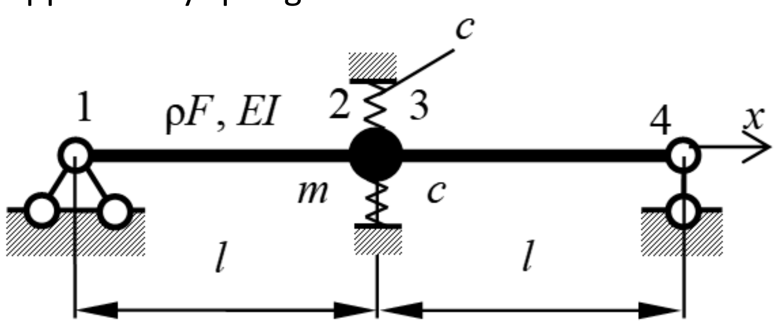
Course: Dynamics of Machines for students learning Applied Mechanics in NRU MPEI, Institute of Power Engineering and Mechanics

Assignment: Find natural frequencies and modes of a hinge-supported beam with a concentrated mass supported by springs



Here:
$2l$ - length of the beam; m - concentrated mass; c - spring stiffness, $c = 3EI/l^3$; ρF , EI - mass per unit length and bending stiffness of the beam.

There are three ways to solve this problem:

1) direct method from non-triviality condition of solution of governing equation
2) initial parameters method
3) finite element method

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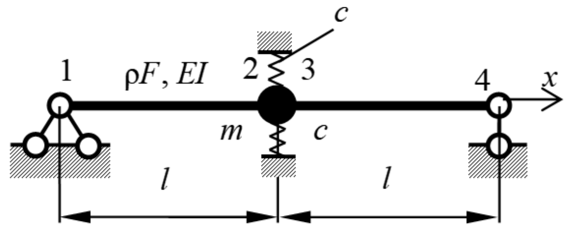
1) Direct method

Governing equations

$$EI\frac{\partial^4 w_1}{\partial x^4}+\rho F\frac{\partial^2 w_1}{\partial t^2}=0,\quad 0<x<l,$$

$$EI\frac{\partial^4 w_2}{\partial x^4}+\rho F\frac{\partial^2 w_2}{\partial t^2}=0,\quad l<x<2l$$



Solution $$w_\alpha(x,t)=\varphi_\alpha(x)\exp(i\omega t),\quad \alpha=1,2$$

Equations for the modes $\varphi_\alpha(\xi)$ $$\frac{d^4\varphi_\alpha}{d\xi^4}-\beta^4\varphi_\alpha=0,\quad \beta^4=\frac{\omega^2\rho F l^4}{EI},\quad \xi=\frac{x}{l}$$

$$\omega_k=\frac{\beta_k^2}{l^2}\sqrt{\frac{\rho F}{EI}}$$

Solution

$$\varphi_1(\xi)=C_1\sin\beta\xi+C_2\cos\beta\xi+C_3\operatorname{sh}\beta\xi+C_4\operatorname{ch}\beta\xi,\quad 0\le\xi\le 1,$$

$$\varphi_2(\xi)=C_5\sin\beta\xi+C_6\cos\beta\xi+C_7\operatorname{sh}\beta\xi+C_8\operatorname{ch}\beta\xi,\quad 1<\xi\le 2$$

Boundary conditions at points 1 and 4 $$\varphi_\alpha=0,\quad \frac{d^2\varphi_\alpha}{d\xi^2}=0,\quad \alpha=1,2$$

Boundary conditions at points 2 and 3 $$\varphi_1=\varphi_2,\quad \frac{d\varphi_1}{d\xi}=\frac{d\varphi_2}{d\xi},\quad \frac{d^2\varphi_1}{d\xi^2}=\frac{d^2\varphi_2}{d\xi^2}$$

+ a jump in transverse force

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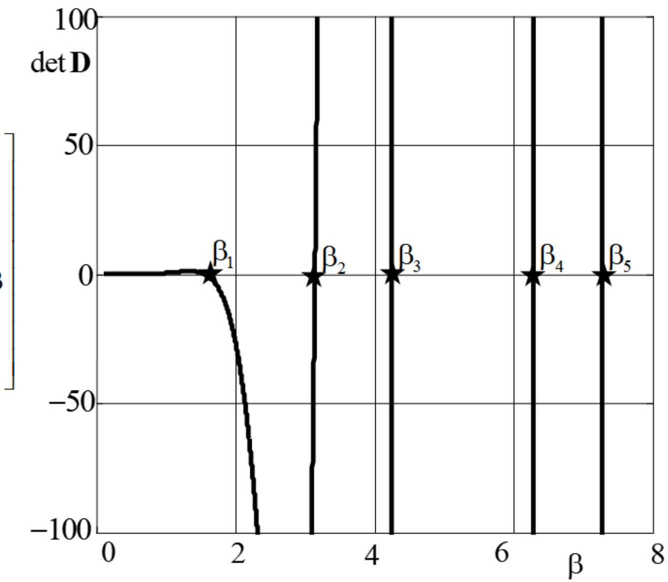
Frequency equation from boundary conditions

$$\det \mathbf{D} = 0$$

$$\mathbf{D} = \begin{bmatrix} \sin\beta & \mathrm{sh}\beta & -\sin\beta & -\cos\beta & -\mathrm{sh}\beta & -\mathrm{ch}\beta \\ \cos\beta & \mathrm{ch}\beta & -\cos\beta & \sin\beta & -\mathrm{ch}\beta & -\mathrm{sh}\beta \\ -\sin\beta & \mathrm{sh}\beta & \sin\beta & \cos\beta & -\mathrm{sh}\beta & -\mathrm{ch}\beta \\ \left(\beta^4-6\right)\sin\beta-\beta^3\cos\beta & \left(\beta^4-6\right)\mathrm{sh}\beta-\beta^3\mathrm{ch}\beta & \beta^3\cos\beta & -\beta^3\sin\beta & -\beta^3\,\mathrm{ch}\beta & -\beta^3\mathrm{sh}\beta \\ 0 & 0 & \sin 2\beta & \cos 2\beta & \mathrm{sh}2\beta & \mathrm{ch}2\beta \\ 0 & 0 & -\sin 2\beta & -\cos 2\beta & \mathrm{sh}2\beta & \mathrm{ch}2\beta \end{bmatrix}$$



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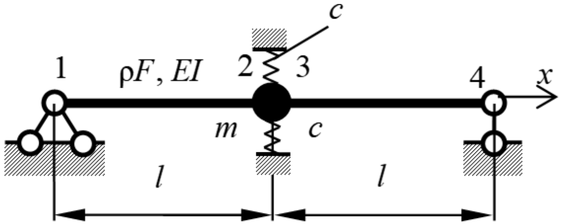
2) Initial parameters method

Mode $$\varphi(\xi) = \varphi_0 S_1(\beta\xi) + \frac{\varphi_0'}{\beta} S_2(\beta\xi) + \frac{M_0}{\beta^2 EI} S_3(\beta\xi) + \frac{Q_0}{\beta^3 EI} S_4(\beta\xi)$$



Initial parameters $\varphi_0,\ \varphi_0',\ M_0,\ Q_0$

$$\mathbf{v}_j = \begin{bmatrix} \tilde{\varphi}_j \\ \varphi_j' \\ \tilde{M}_j \\ \tilde{Q}_j \end{bmatrix}, \quad \tilde{\varphi}_j = \frac{\varphi_j}{l}, \quad \tilde{M}_j = \frac{M_j l}{EI}, \quad \tilde{Q}_j = \frac{Q_j l^2}{EI}$$

$$j = 1, 2, 3, 4.$$

$$\mathbf{v}_j = \mathbf{A}_{j,j+1}\mathbf{v}_{j+1}$$

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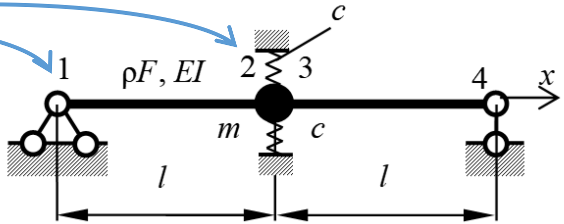
2) Initial parameters method



Transition matrices $\mathbf{v}_j = \mathbf{A}_{j,j+1}\mathbf{v}_{j+1}$, j – point's number

$$\mathbf{A}_{12} = \mathbf{A}_{34} = \begin{bmatrix} S_1(\beta) & \frac{1}{\beta}S_2(\beta) & \frac{1}{\beta^2}S_3(\beta) & \frac{1}{\beta^3}S_4(\beta) \\ \beta S_4(\beta) & S_1(\beta) & \frac{1}{\beta}S_2(\beta) & \frac{1}{\beta^2}S_3(\beta) \\ \beta^2 S_3(\beta) & \beta S_4(\beta) & S_1(\beta) & \frac{1}{\beta}S_2(\beta) \\ \beta^3 S_2(\kappa_1) & \beta^2 S_3(\kappa_1) & \beta S_4(\beta) & S_1(\beta) \end{bmatrix}. \qquad \mathbf{A}_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 6-\beta^4 & 0 & 0 & 1 \end{bmatrix}$$

Then $\mathbf{v}_2 = \mathbf{A}_{12}\mathbf{v}_1$, $\mathbf{v}_3 = \mathbf{A}_{23}\mathbf{v}_2$, $\mathbf{v}_4 = \mathbf{A}_{34}\mathbf{v}_3$ and $\mathbf{v}_4 = \mathbf{A}_{34}\mathbf{A}_{23}\mathbf{A}_{12}\mathbf{v}_1$.

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2) Initial parameters method

$$\mathbf{v}_4 = \mathbf{A}_{34}\mathbf{A}_{23}\mathbf{A}_{12}\mathbf{v}_1.$$

Boundary conditions

$$\begin{bmatrix} 0 \\ \varphi_4' \\ 0 \\ \tilde{Q}_4 \end{bmatrix} = \mathbf{A}_{34}\mathbf{A}_{23}\mathbf{A}_{12}\begin{bmatrix} 0 \\ \varphi_1' \\ 0 \\ \tilde{Q}_1 \end{bmatrix}, \quad \varphi_1'\mathbf{f}(\beta) + Q_1\mathbf{g}(\beta) = \begin{bmatrix} 0 \\ \varphi_4' \\ 0 \\ \tilde{Q}_4 \end{bmatrix};$$

where $\mathbf{f}(\beta) = \mathbf{A}_{34}\mathbf{A}_{23}\mathbf{A}_{12}\mathbf{m}, \quad \mathbf{g}(\beta) = \mathbf{A}_{34}\mathbf{A}_{23}\mathbf{A}_{12}\mathbf{n}$

$$\mathbf{m}' = [0 \quad 1 \quad 0 \quad 0], \quad \mathbf{n}' = [0 \quad 0 \quad 0 \quad 1].$$

$$\Delta(\beta) = f_1(\beta)\,g_3(\beta) - f_3(\beta)\,g_1(\beta) = 0$$

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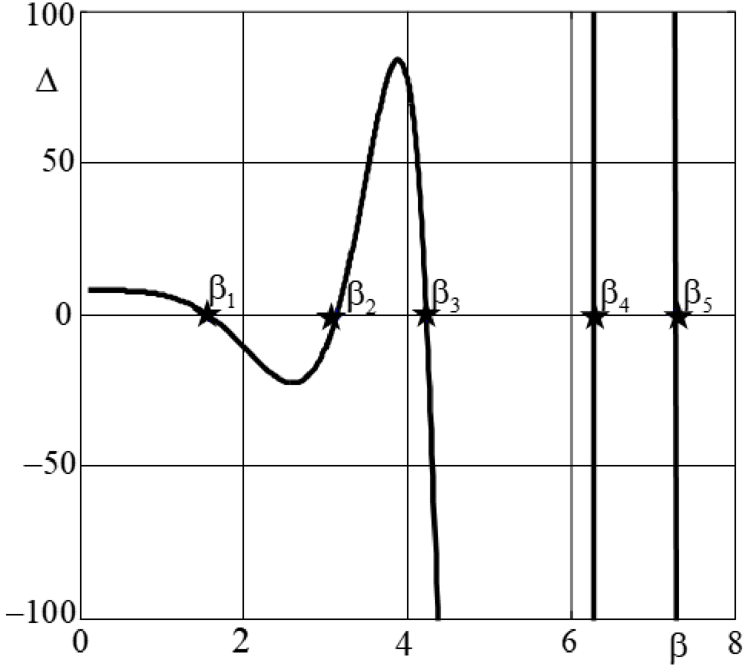
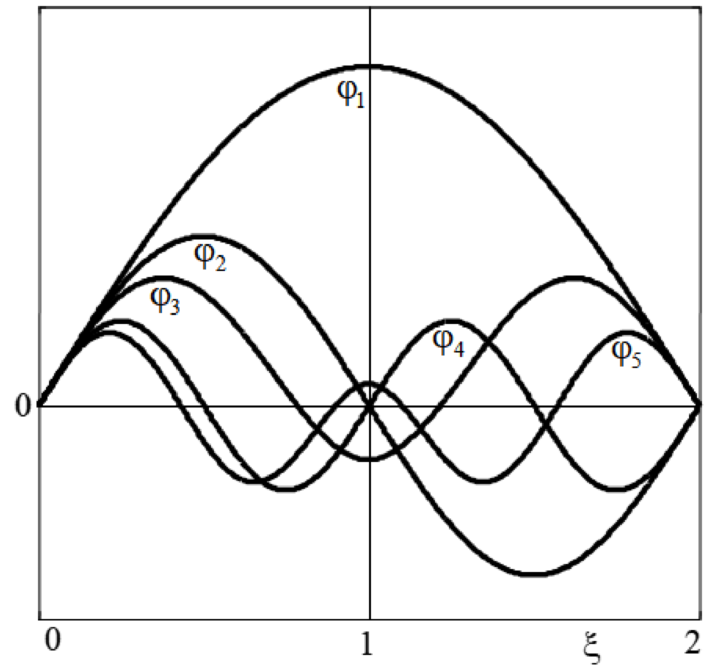
2) Initial parameters method





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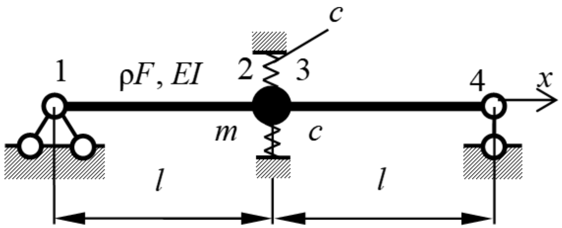
3) Finite element method

$$\mathbf{K} = \frac{2EI}{a^3}\begin{bmatrix} 6 & 3a & -6 & 3a \\ 3a & 2a^2 & -3a & a^2 \\ -6 & -3a & 6 & -3a \\ 3a & a^2 & -3a & 2a^2 \end{bmatrix} \qquad \mathbf{M} = \frac{\rho F a}{420}\begin{bmatrix} 156 & 22a & 54 & -13a \\ 22a & 4a^2 & 13a & -3a^2 \\ 54 & 13a & 156 & -22a \\ -13a & -3a^2 & -22a & 4a^2 \end{bmatrix}$$

$$\left|\mathbf{K} - \omega^2\mathbf{M}\right| = 0 \qquad [\omega^2, \mathbf{V}] = \mathrm{eig}(\mathbf{K}, \mathbf{M})$$



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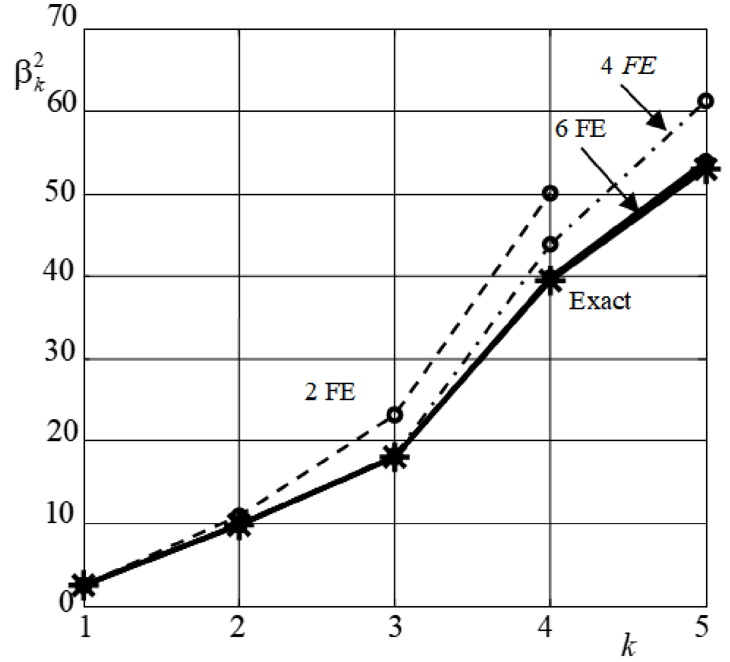
3) Finite element method



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Thank you for attention!

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