The Advantages of Using Computer Algebra System Maple in Learning Differential Equation

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Mathematics as a classic subject is vital in engineering education. The emergence of mathematical IT programs (MathCad, MatLab, Matematica, Maple) requires a revision of traditional methods of teaching mathematics.

Using math software to support the teaching of mathematics in engineering education opens up new possibilities.
Algorithm for solving typical problems can serve as an instrument for cognition, control, and development of one’s own knowledge of mathematics.

There are opportunities such as
✓ the visualization,
✓ animated images designed.
CAS Maple has been advantageously used for modeling and for solving recurrence equations, for analytical researches in theory of mechanical equilibrium, for studying linear transformations.

CAS Maple features are for teaching different types of function approximating techniques, such as interpolation-, least square-, spline and uniform approximation methods. Visual computing is used in interactive (on-line) teaching the principal divisions of mathematical physics.
The excellent graphical capabilities of CAS Maple allow students to learn several methods for solving a problem, introducing elements of the experiment and scientific research into their study.
The authors present the experience of using CAS Maple when studying the section **Differential Equations**.

A class of integral curves defines the general solution of the ordinary differential equation (ODE). Each of the integral curves corresponds to some parameter points, i.e. constants C.

The capabilities of CAS Maple allow not only perform general problem solving ODE, but also to map the resulting class of integral curves, specifying specific values of one or another finite number of parameters.
Example 1.

On integrating DE with separable variables

\[ \cos y \, dy + \sin^3 y \, e^x \, dx = 0 \]

provided that \( \sin y \neq 0 \) one perform general solutions using the \textit{dsolve} command and map the obtained functions in one coordinate system.
It should be noted that in this example, DE has also got special solutions, i.e. as functions of

\[ y = \pi k, \; k \in \mathbb{Z} \]

that are obtained when considering the case \( \sin y = 0 \).

A peculiarity of the partial differential equation (PDE) is that it is required here to specify not some parameter values but some functions for the unique performing of a particular solution.
Example 2.

For a linear inhomogeneous DE

\[ u = y - x + f \left( y^2 - x^2 \right) \]

where \( f = f(x) \) is complementary function.

For visual reference of a particular solution for various \( F1 \) functions, one needs to set some function and use the \textit{plot3d} command to plot a three-dimensional diagram.

\[ F1 = x \]

\[ F1 = \sin(x) \]
Example 3 - example illustrates geometry treatment of Cauchy problem for first-order PDE.

Define a surface area passing through a right line $y = x, z = 1$ and orthogonal right-angled to surface areas

$$S : x^2 + y^2 + z^2 = Cx$$

Having completed the necessary transformations, the problem is reduced to finding a particular solution of the obtained DE in partial derivatives that satisfies the conditions $y = x, z = 1$.

Solving a given equation by the method of characteristics, one performs a particular solution - the equation of the required surface area

$$2y^2 + z^2 = \left( x^2 + y^2 + z^2 \right) z$$
According to the condition, the found surface must be orthogonal to spherical surfaces

\[ S : x^2 + y^2 + z^2 = Cx \]

An idea of the way these surfaces intersect can be obtained by plotting them in one coordinate system.
In addition to static images, CAS Maple has the capability to design animation. The *liveliness/revitalization* of the process enhances the effect of modeling a phenomenon greatly.

Design an animated video clip showing the vibrations of a string is an example of assignment from one of the laboratory works that students perform.
While studying numerical techniques for solving differential equations, the authors also use CAS Maple. Here are some examples of lab assignments that students do.

**Example 4.** Investigate the issue of the best choosing the number of collocation points to perform an approximate answer to a linear differential equation that fulfill given boundary/edge conditions. Compare the approximate answer found by the collocation method to the exact/accurate solution of the equation.

Example 6. Using the mesh method, solve the mixed problem for the heat conduction equation. Display the temperature/heat distribution in the centre piece at different time intervals.

Example 7. Using the mesh method, solve the mixed problem for the string oscillation equation. Depict the change in the string profile at different time intervals.
The experiment, which was conducted at Sholom-Aleichem Priamursky State University, Pacific National University, Moscow State University of Civil Engineering and Naryn State University in 2017-2018 and 2018-2019 academic years, allowed the authors to estimate the advantages of using CAS Maple for learning differential equations.
Results

For first-year control group consisted of 124 people were held in the regular way. For experimental group involved 133 people CAS Maple was used in the classroom. Both groups are comparable to each other in performance: according to the findings of the first examination period, the quality performance in the groups was 45% and 47%, respectively.
After the students had studied *Differential Equations* section, both groups of them took a test containing two types of tasks: 1) the reproductive level of knowledge and skills, 2) mastering research skills. The students’ outcomes of the test in both groups are presented in Table. Accomplishment of tasks of the reproductive level in the experimental group is 16% more. Moreover, the per cent in the number of students who displayed a mastery of research skills is way above in the experimental group.

<table>
<thead>
<tr>
<th>Group</th>
<th>Task level</th>
<th>Test task performance</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>“5” and “4”</td>
<td>“3”</td>
</tr>
<tr>
<td>Test</td>
<td>1</td>
<td>41.1 %</td>
<td>50.8 %</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>27.4 %</td>
<td>33.1 %</td>
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<tr>
<td>Experimental</td>
<td>1</td>
<td>57.9 %</td>
<td>41.3 %</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>50.4 %</td>
<td>35.3 %</td>
</tr>
</tbody>
</table>
The findings of the study allow the authors to conclude that learning using CAS Maple has advantages over conventional learning. Students increase motivation, master creative autonomy, and develop research skills.
Thank you for attention!

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