Model of Piezoelectric Transducers for the Metrological Characteristics Study of Ultrasonic Measuring Instruments

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INTRODUCTION

- The use of IT technologies in the educational process and scientific laboratories can significantly expand the capabilities of a laboratory workshop in the measuring transducers study, combining full-scale experiments with studies of primary measuring transducers (sensors) on their models.
- Most of the sensors can be represented in the form of equivalent electrical circuits and investigated using well-known programs for electrical circuits simulation (OrCAD, PSpice, MatLab Simulink). Students learn directly the sensors mathematical models, their device and design, the operation principle, and they master the skills of research and development of complex measuring instruments using modern IT technologies.
- If approached from the standpoint of measuring technology, well-known models often require additional research and refinement, and therefore cannot be used in their existing form when developing the measuring devices design.
- In the framework of this presentation, it is proposed to consider in detail the issue of developing a piezoelectric transducer model (PZT), which is an integral part of an ultrasonic liquid flowmeter (UFM) in order to study its metrological characteristics. The results obtained for this given example can be useful in the development of other diagnostic and measuring ultrasonic devices.

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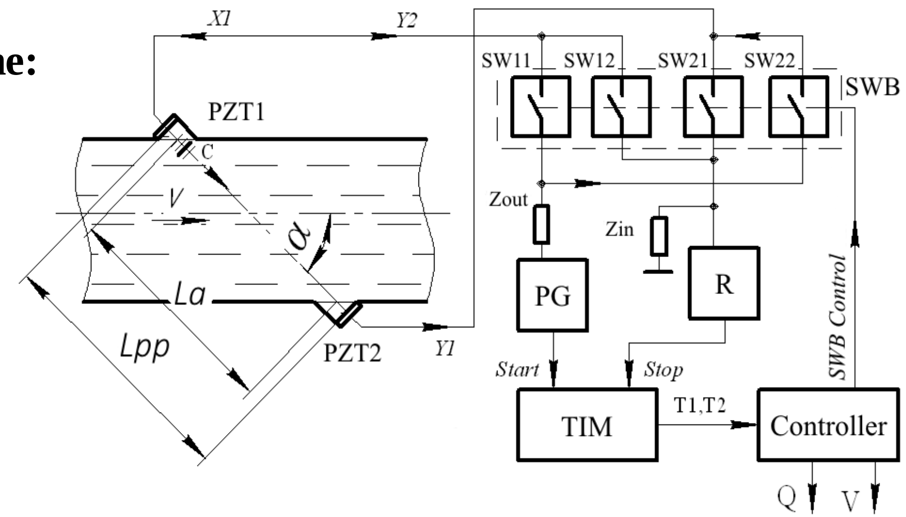
THE UFM OPERATION PRINCIPLE

The generalized time-difference UFM scheme:

Figure explanations: SWB is a signal switchboard (emitter direction switch); SWij — individual switching keys of SWB; PG — transmitted (emitted) pulses generator (impulse driver); R — receiving device (receiver); TIM is a time intervals meter).



The UFM transfer (conversion) function:

$$\Delta T = T_2 - T_1 = \frac{L_a}{C^2} v + \Delta t_a$$

where $v = K_g Q / S$; ΔT – the UPS propagation times difference in the T_1, T_2 – the UPS propagation times, respectively, along the fluid flow and against the fluid flow; L_{PP}, L_a – the distance between the PZT emitting surfaces, and the active part of the ultrasonic signal path at which the fluid velocity is nonzero; C – the ultrasound speed in a stationary fluid; α – the angle of the ultrasonic wave propagation in the pipeline; v – the average fluid velocity along the ultrasound beam length; Q – the measured fluid flow rate (flow of the velocity vector v through the pipeline cross-section S); K_g – the hydrodynamic correction coefficient, taking into account the relationship of the measured v with the average fluid velocity over the pipeline cross-section; Δt_a – the additional difference in the propagation times of the signals Y1 (along the fluid flow, from PZT1 to PZT2) and Y2 (against the fluid flow), caused by their unequal delay time in the flowmeter EAP, the so-called "*EAP asymmetry*".

THE PROBLEM STATEMENT

- The error in measuring the times T_1 and T_2 is largely determined by the signal-to-noise ratio in the received signals, therefore, when PZT developing, its try to maximize the modules of the UFM electro-acoustic path (EAP) end-to-end transfer functions:

$$W(j\omega)_{12} = W_{PE1E} W_{PE2R} A(j\omega) e^{-j\omega T_a}$$

$$W(j\omega)_{21} = W_{PE2E} W_{PE1R} A(j\omega) e^{-j\omega T_a}$$

where $W_{PEiE} = \frac{p_{Ei}(j\omega)}{U_i(j\omega)}$, $W_{PEiR} = \frac{U_{Ri}(j\omega)}{p_{Ri}(j\omega)}$; $W(j\omega)_{12}$, $W(j\omega)_{21}$ – "end-to-end" ("through") transfer functions of the EAP during signal propagation, respectively, along the flow and against the fluid flow; W_{PEiE}, W_{PEiR} – transfer functions of the i-th PZT in the emission and reception mode; $A(j\omega)$ – a coefficient characterizing the additional signal delay caused by diffraction of a narrow ultrasound beam; T_a – the signal delay time in the acoustic path, the same for both wave probing directions.

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THE PROBLEM STATEMENT (continuation)

- The EAP asymmetry is one of the UFM measurement error. For decreasing the asymmetry in modern flowmeters the following hardware features are used:
  - a common pulse generator and a signal receiver are used for both wave emission directions. This ensures maximum identity of the conditions for the propagated signals and minimizes the value Δt_a – the additional difference in the signals propagation times;
  - the PG output impedance and the receiver R input impedance should be the same for signals in both directions.
- A difference in the PG output and the R input resistances leads to a change in the filling ultrasonic signals frequency excited upstream and downstream. This can be explained by the difference between the end-to-end transfer functions $W(j\omega)_{12}$ and $W(j\omega)_{21}$, which occurs when each of the PZTs is switched from emission to reception.
- The reciprocity theorem is fulfilled for acoustic media.
- The main reason for the reciprocity principle violation will be the possible differences in the each PZT transfer functions for emission and reception: $W_{PE1E} \neq K_1 W_{PE1R}$ or $W_{PE2E} \neq K_2 W_{PE1R}$.

THE PZT MATHEMATICAL MODEL

- Constructively, the PZT, in the simplest case, contains a flat piezoelectric element (PE) oscillating in thickness direction (longitudinal piezoelectric effect).
- The PZT electromechanical model appears to be a linear (and, therefore, reversible) four-terminal device, the coefficients of which are found from the piezoelectric effect equations and the PE parameters.

System of equations determining the state of piezoceramics:

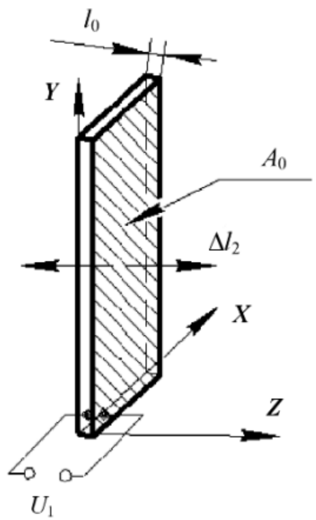
$$D_3 = \varepsilon_{33}^{S} E_3 + e S_3$$

$$T_3 = -e E_3 + c_{33}^{E} S_3$$

where D_3 – the Z-component of the PE polarization vector; ε^{S}_{33} – dielectric constant of piezoceramics in the Z-axis direction, [F/m]; E_3 – Z-component of the electrical tension vector; e_{33} – piezoelectric constant, [C/m^2]; S_3 – relative deformation; c^{E}_{33} – the elastic modulus of the PE material in the absence of an electric field (for example, with closed PE electrodes).

Figure explanations: l_0 – the plate thickness, A_0 – the plate area, U_1 – the voltage applied to the PE electrodes, Δl_2 – the displacement of the plate face sides.

A piezoelectric element that performs longitudinal oscillations (vibrations) in thickness:



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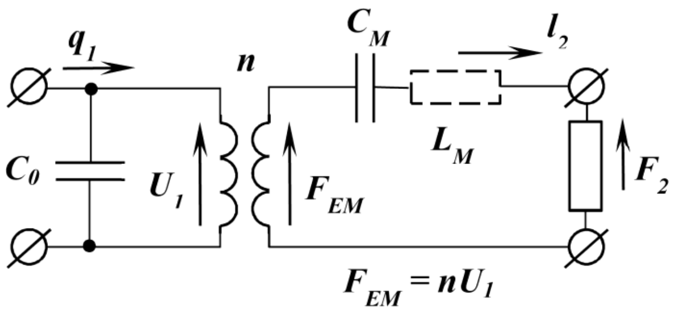
THE PZT MATHEMATICAL MODEL (continuation)

- Believe that the electric field and mechanical stresses are uniformly distributed over the thickness of a flat PE, respectively, the inertia (and kinetic energy) of its elementary volumes can be neglected:

Equivalent PE circuit when operating at low frequencies:



$$q_1 = DA_0 = \left(\varepsilon_{33}^S \frac{U_1}{l_0} + e_{33} \frac{\Delta l_2}{l_0} \right) A_0$$

$$F_2 = T_3 A_0 = \left(-e_{33} \frac{U_1}{l_0} + c_{33}^E \frac{\Delta l_2}{l_0} \right) A_0$$

or in a more compact form:

$$q_1 = C_0 U_1 + n \Delta l_2$$

$$F_2 = -nU_1 + K_\Delta^E \Delta l_2$$

$$n = e_{33} \cdot \frac{A_0}{l_0} \qquad C_0 = \varepsilon_{33}^S \frac{A_0}{l_0} \qquad C_M = 1/K_\Delta^E$$

where q_1, U_1 – the electric charge and voltage at the PE electrodes are the input values of the electromechanical four-terminal network; Δl_2, F_2 – the displacement of the PE face sides relative to each other and the force acting from the PE side on the external environment is the output values of the electromechanical four-terminal network; n – electromechanical transformation coefficient (CEMT), [C/m]; C_0 – PE capacity in the inhibited state; C_M – mechanical flexibility; F_{EM} – the electromechanical force.

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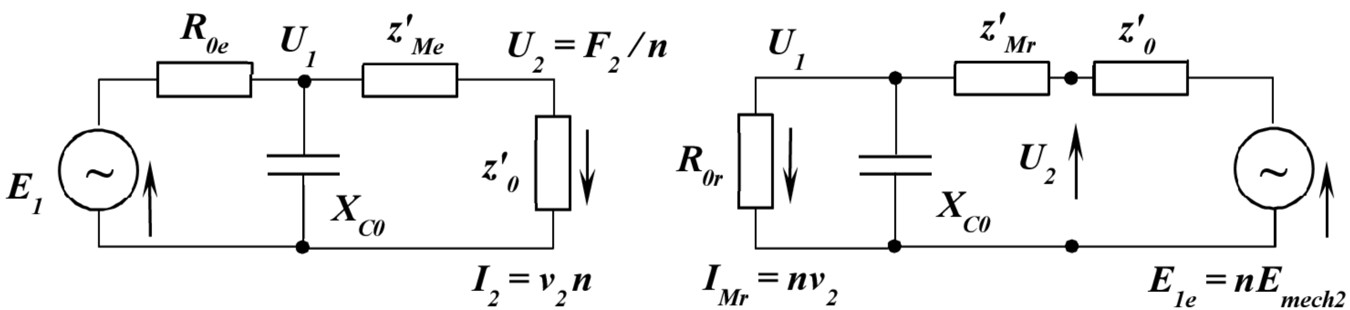
THE PZT MATHEMATICAL MODEL (continuation)

- Simplified equivalent PE circuits for harmonic signals near the resonant frequency: in the emission mode (see left Fig., $F_2 = 0$) and in the reception mode (right Fig., $E_1 = 0$ – the generator voltage).



$$z'_M = \left(X_{C_M} + X_{L_M}\right) / n^2$$

$$z'_0 = z_0 / n^2$$

- emission ($E_{mech2} = 0$):

$$U_1 = E_1 - R_{0e} I_1$$

$$F_2 = v_2 z_0$$

- reception ($E_1 = 0$):

$$U_1 = -z_0 I_1$$

$$F_2 = E_{mech2} - v_2 z_0$$

According to the electromechanical analogy principle, the elements z'_{Me}, z'_{Mr} – the PE mechanical impedances in the emission and reception modes, reduced to the electrical part, and z'_0 – the reduced wave resistance of the medium. E_{mech2} – mechanical driving force; R_{0e} – resistance R_0 in the emission mode (internal generator resistance), [Ω], further denote it Rg, and in the reception mode – Rr; z_0 – the wave resistance of the medium for a flat PE, [N·s/m], $z_0 = \rho_2 c_2 A_0$; ρ_2, c_2 – emission medium density and speed of ultrasonic waves in medium, respectively; P_2 – the acoustic pressure in the incident wave.

$$E_{mech2} \approx 2 P_2 A_0$$

INFLUENCE OF ELECTRICAL PARAMETERS ON THE ELECTRO-ACOUSTIC PATH ASYMMETRY

- The reciprocity principle is satisfied if $z'_{Me} = z'_{Mr}$. This equality may be violated if during reception and emission, the electric modes of the PE operation are different.
- The theoretical assessment of the receiver R_r and the generator R_g resistances influence on the PE elastic modulus:

$$F_2 = \left[\left(e_{33} n \frac{\omega^2 R_\Pi \tau_0}{(\omega\tau_0)^2 + 1} - e_{33} n \frac{j\omega R_\Pi}{(\omega\tau_0)^2 + 1} \right) \frac{2}{\pi^2} + c_{33}^E \right] \frac{\pi^2 A_0}{2 j\omega l_0} + j\omega \cdot m_э v_2$$

In the given expression, the term in brackets is the equivalent elastic modulus of piezoceramics in the half-wave vibrator mode: a term depending on the receiver resistance is added to the elastic modulus c^E_{33}:

$$\Delta c_{33}^{R_r} = \left(e_{33} n \frac{\omega^2 R_r \tau_0}{(\omega\tau_0)^2 + 1} - e_{33} n \frac{j\omega R_r}{(\omega\tau_0)^2 + 1} \right) \frac{2}{\pi^2}$$

- If the receiver R_r and the generator R_g resistances are exactly equal to each other, then the reciprocity theorem will be fulfilled and the EAP asymmetry will be absent.

Limiting cases: when $R_r \to 0$ the elastic modulus tends to c^E_{33}, while $R_r \to \infty$ the elastic modulus tends to c^D_{33}. At intermediate values of R_r, the elastic modulus has a frequency dependent complex value (the ultrasound speed and the wave number will also be complex).

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THE EXPERIMENT PROCEDURE SCHEME

The electro-acoustic path prototype model made at the stand for studying the influence of the electro-acoustic path parameters on the received signals shape and its asymmetry:

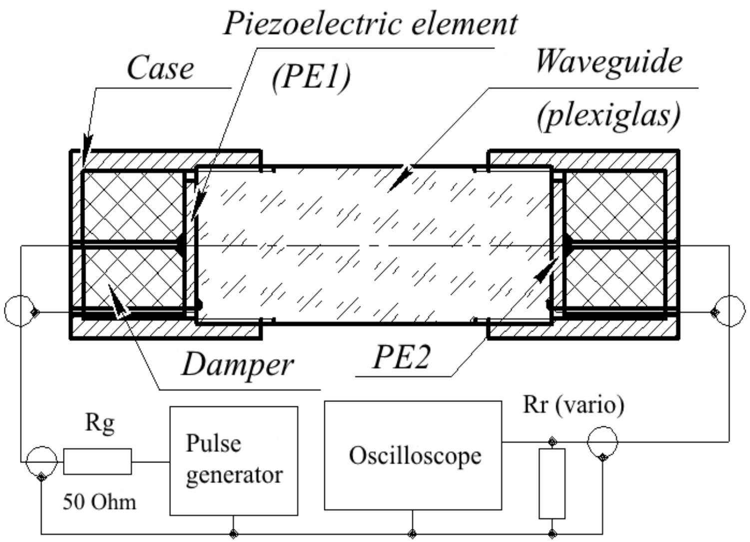

Figure explanations and design features:

- two piezoelectric elements PE1 and PE2 coaxially attached to the waveguide (sound conductor) using damping pads – coaxially mounted on the waveguide made of organic glass (plexiglas);
- to improve the acoustic contact between PE and the waveguide, the contact layer – liquid lubricant was used (layer thickness did not exceed than 5 μm);
- PE is preloaded by means of the PZT threaded connection to the waveguide, while dampers are pressed to the PE reverse sides;
- the collapsible constructive device made it possible to vary the parameters of EAP elements.

Features and conditions of the experiment:

- the PE was excited from a PG, the received signals were connected to an oscilloscope with an input impedance of 10 MΩ;
- the waveguide length was chosen sufficient so that the propagation time was longer than the emitted signals duration to exclude the influence of piezoelectric elements on each other;
- the PE excitation was carried out by a radio-pulse with a variable filling frequency (the number of full periods is 12) to reduce the emitted signals duration when taking the amplitude-frequency characteristic.

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THE EXPERIMENT PROCEDURE SCHEME (continuation)

The EAP equivalent scheme (for two directions of UPS propagation):

- The equivalent circuit contains two EAP models:
  - one is the reference with the nominal resistances values R_g and R_r;
  - the other with the changed resistances values R_g or R_r (and the changed capacitance values C1).

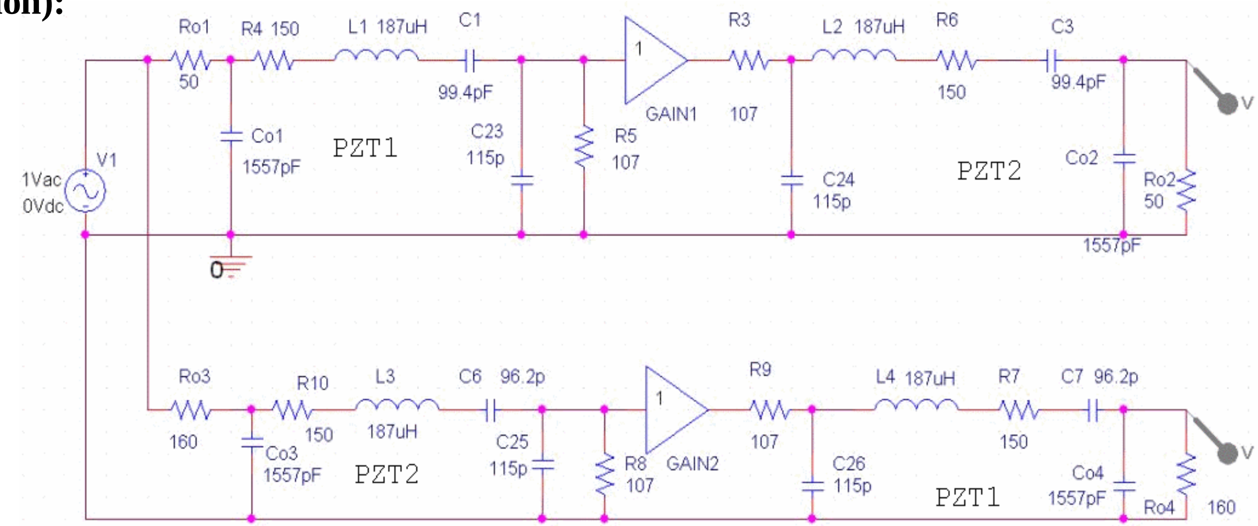

Figure explanations: resistances R4, R6, R7, R10 simulate PZT dampers, which are the PE acoustic load; the resistances R5, R3, R8, R9 are equivalent to the wave impedance of an extended waveguide; capacitors C23, C24, C25, C26 model the contact layer of "PE – waveguide“; the resistances Ro1, Ro3 characterize the internal resistances R_g of the signal source — generator; Ro2 and Ro3 – the receiver resistances (R_r). In the asymmetry study, only Ro4 changes and the corresponding capacitance C1.

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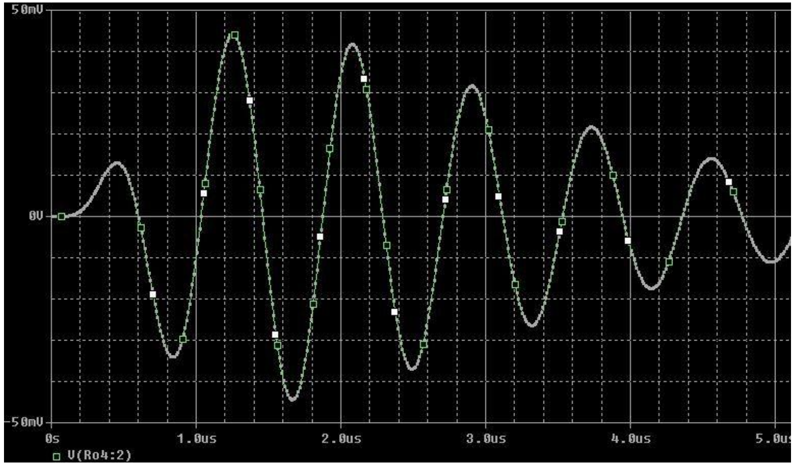
THE EXPERIENCE CONDITIONS. SIMULATION RESULTS

The received signal graph according to the OrCAD simulation results:



- Characteristic parameters of the EAP frequency response:
  - $W(f_o)$ – the AFC module at the maximum point;
  - f_o – the AFC maximum frequency (resonance frequency);
  - $\Delta f_{0.7}$ – the bandwidth at the 0.7 level.

Table I. Baseline PZT data without a membrane (initial parameters for piezoelectric elements calculating):

D_{PE}, [m]	ρ, [kg / m^3]	A_0, [m^2]	l_0, [m]	m_{PE}, [kg]
0.02	7740	0.000314	0.0015	0.003647
e_{33}, [C/m^2]	c^E_{33}	ε^S_{33}, [F/m]	C^E_1, [F][a]	L_1, [H]
14.9	$9.3\cdot10^{10}$	$7.43\cdot10^{-9}$	$1.01\cdot10^{-10}$	$1.87\cdot10^{-4}$

- Notes and features:
  - C^E_1 – the equivalent value of the capacitance C_1 with the generator resistance $Rg = 0$;
  - PE is made of "ЦТС-19" piezoceramics (worldwide analog is PZT-5) with 20 mm in diameter and a nominal thickness of 1.5 mm;
  - a short pulse generator is used to take the impulse response (upon PZT excitation with pulses of $t_{imp} = 0.1$ μs for $R_r = R_g = 160$ Ohm).

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THE EXPERIMENTAL RESULTS

The received signal graph according to the experiment results:

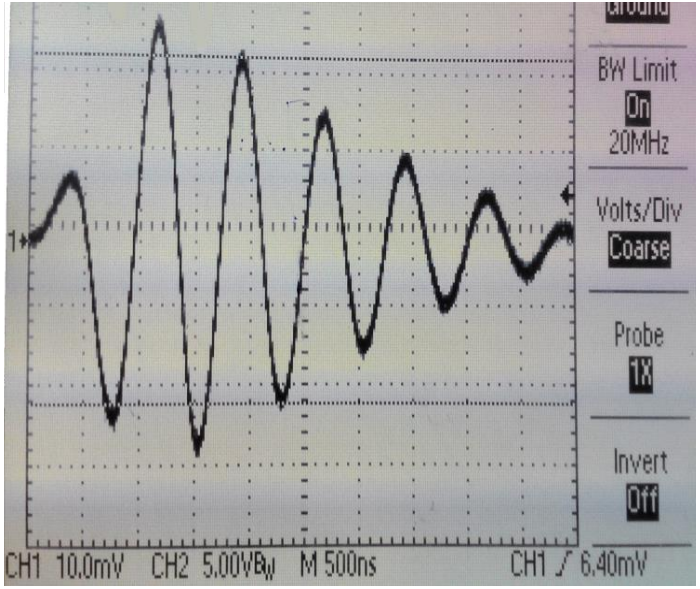

Table II. Calculation of the capacitances values C1, taking into account the R_r or R_g resistances influence:

Rr = = *Rg*, [Ω]	The Mathematical Model			The Prototype Model		
	$W(f_o)$	f_o, kHz	$\Delta f_{0.7}$, kHz	$W(f_o)$	f_o, kHz	$\Delta f_{0.7}$, kHz
50	0.0462	1178.5	165.2	0.023	1250	180.0
160	0.0435	1210.7	171.7	0.021	1285	185.0

- The identity of the EAP theoretical and real models was assessed by the following parameters of the received signals:
  - by the EAP asymmetry value with a change in the resistance R_r;
  - by changing the EAP amplitude-frequency characteristics while simultaneously changing the R_r and R_g resistances ($R_r = R_g$).

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Analysis of the achieved research results:

- The difference in resonant frequencies can be explained by the difference between the real PE parameters from the nominal values. The difference in resonant frequencies is obtained due to their dependence on the electromechanical transformation coefficient and the transformation coefficient. Determination of the real PE resonant frequency is extremely important, it can vary by 30-40 % depending on the R_g and R_r.
- The PE resonant frequency substantially depends on the R_g and R_r resistances. This is the cause of the EAP asymmetry. A change in the R_r resistance by 10 % of the nominal value (R_g – const) leads to a change in the signal propagation time in the EAP by (4 – 4.5) ns.

CONCLUSIONS

- The proposed model of flat piezoelectric elements using the longitudinal piezoelectric effect can be used to determine:
  - the optimal parameters of piezoelectric transducers;
  - the requirements for the generator's output circuits and the receiver's input circuits in the design of ultrasonic measuring instruments.
- The calculated and experimental data on the EAP asymmetry study on the mathematical model and the prototype model showed good agreement between the results, that confirms the adequacy of the proposed EAP model. The EAP model is very close with the real EAP for ultrasonic flowmeters with non-contact (clamp-on) PZTs.
- Studying the proposed model as part of laboratory work on measuring transducers will allow students to better learn the teaching material and master the skills of working with modern computer-aided design systems.

Thank you for attention!

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